PHSX 212 — Final Exam — December 14, 2006

1. This exam consists of 18 problems plus a bonus one. Problems 1 to 9 will be graded by scantron and problems 10-18 as well as the bonus problem (Problem 19) will be graded by hand. Each scantron question has only one correct answer. In order to complete 19 questions in 2.5 hours you need to spend an average of 6-7 minutes on each problem. The formula sheet and scratch paper are located at the end of the exam.
2. All marks on the answer sheet should be #2 black lead pencil.
3. Do not start doing any problems before filling out the ID section on the answer sheet. Be sure to fill in the dots below your name and ID number.
4. Having more than one answer sheet in your possession or in your general vicinity is forbidden and will be treated as academic misconduct.
5. If you are not 100% sure about the correct answer to a question, you should still choose one of the answers. Wrongly answered questions incur no penalty.

CHECK THAT YOU HAVE COMPLETED THE FOLLOWING:

Name entered left-justified on answer sheet.  
Student ID entered left-justified on answer sheet.

For grading use:

<table>
<thead>
<tr>
<th>Problem</th>
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<tr>
<td>Subtotal 1-9 (40 possible)</td>
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<td>Bonus problem (10 possible)</td>
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<td>Total (110 possible)</td>
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1. (4 points) A light wave has an electric field given by \( E = 9 \text{V/m} \sin(\text{kx} - \omega t) \), pointing in the \( x \) direction. Find the magnitude of the magnetic field \( B_m \) associated with the light wave and the intensity \( I \) of the energy transmitted by this wave.

(a) \( B_m = 6 \times 10^{-8} \text{T} \) and \( I = 0.28 \text{ J m}^{-2} \text{s}^{-1} \)
(b) \( B_m = 6 \times 10^{-8} \text{T} \) and \( I = 0.93 \text{ J m}^{-2} \text{s}^{-1} \)
(c) \( B_m = 3 \times 10^{-8} \text{T} \) and \( I = 0.11 \text{ J m}^{-2} \text{s}^{-1} \)
(d) \( B_m = 3 \times 10^{-8} \text{T} \) and \( I = 0.84 \text{ J m}^{-2} \text{s}^{-1} \)
(e) \( B_m = 1 \times 10^{-8} \text{T} \) and \( I = 0.69 \text{ J m}^{-2} \text{s}^{-1} \)

\[
\frac{E}{B} = \frac{c}{2\mu_0}
\]

2. (4 points) A railroad train is traveling at high speeds in still air. The frequency of the note emitted by the train whistle is 300 Hz. What frequency is heard by a passenger on a train moving in the opposite direction to the first at 15 m/s approaching the first train? The speed of the sound is 343 m/s.

(a) 354.5 Hz
(b) 278.1 Hz
(c) 249.8 Hz
(d) 313.7 Hz
(e) 300 Hz

3. (4 points) Which of the following actions will increase the electric field strength at point P?

(a) Make the rod longer without changing the charge.
(b) Make the rod shorter without changing the charge.
(c) Move the rod down without changing the charge.
(d) Move the rod further away without changing the charge.
(e) Remove charge from the rod.
4. (4 points) To what electrical potential should you charge a 1.0 \( \mu \)F capacitor to store 1.0J of energy?

(a) 1V  
(b) 1 V/m  
(c) 1.4V  
(d) 2V  
(e) 0.71V

5. (6 points) A 15 meter long power transmission cable is made from a 10 meter long Al wire joined with a 5 meter long Cu wire, as shown in the figure. The diameters of the Al and Cu wires are both 1.2 mm. Calculate the resistance of the 15 meter long cable. \( \rho_{Al} = 2.65 \times 10^{-8} \) \( \Omega \) m and \( \rho_{Cu} = 1.68 \times 10^{-8} \) \( \Omega \) m

(a) 0.071 \( \Omega \);  
(b) 0.0037 \( \Omega \);  
(c) 10.14 \( \Omega \);  
(d) 1.78 \( \Omega \);  
(e) 0.31 \( \Omega \).

\[
R = \frac{\rho L}{A} = \frac{2.65 \times 10^{-8}}{10} = 2.65 \times 10^{-8} \Omega
\]

6. (4 points) An 8.0 meter long copper wire is carrying a current of \( I = 6.5 \) A. If the electric potential difference from end to end is 15.0 V, calculate the electric power in the wire.

(a) \( P = 15.7 \) Joule  
(b) \( P = 27.3 \) Joule  
(c) \( P = 97.5 \) Joule  
(d) \( P = 6430.7 \) Joule  
(e) \( P = 1482.9 \) Joule

\[
P = I^2 R = \Delta V = I R
\]

\[
P = I \Delta V
\]
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A (4 points) Electrons are traveling from left to right along the tapered wire shown below. They enter from the left with a drift velocity \( V_{in} = 8.0 \times 10^{-4} \text{ m/s} \). The radius at the left of the wire is \( R_{in} = 1.0 \text{ mm} \) and the radius at the right is \( R_{out} = 2.0 \text{ mm} \). What is the magnitude of the drift velocity and direction of the electrical current at the right end?

\[ I = y_1 v_{drift} = n e A z v_d \]

(a) \( V_{out} = 5 \times 10^{-4} \text{ m/s} \) and the electrical current is from left to right
(b) \( V_{out} = 4 \times 10^{-4} \text{ m/s} \) and the electrical current is from right to left
(c) \( V_{out} = 3 \times 10^{-4} \text{ m/s} \) and the electrical current is from left to right
(d) \( V_{out} = 2 \times 10^{-4} \text{ m/s} \) and the electrical current is from right to left
(e) \( V_{out} = 1 \times 10^{-4} \text{ m/s} \) and the electrical current is zero

8. (6 points) An electron traveling at a velocity of \( \vec{v} = \left( 4 \times 10^5 \text{ m/s} \right) \) enters the region between two capacitor plates as shown below. The plates are located a distance \( d = 1 \text{ mm} \) apart and are connected to a 400 V battery. What is the magnetic field that needs to be supplied to the region between the plates to keep the electron from being deflected as it passes through the capacitor?

(a) \(-0.10 \text{ T} \hat{k}\)  
(b) \(+0.10 \text{ T} \hat{k}\)  
(c) \(-0.25 \text{ T} \hat{k}\)  
(d) \(+0.25 \text{ T} \hat{k}\)  
(e) \(+0.40 \text{ T} \hat{k}\)

\[ E = V \times B \]
\[ \frac{\Delta}{\hat{z}} = \text{into port} \]
\[ E = VB \text{ Sum B} \]

9. (4 points) An LRC series circuit has \( L = 0.6 \text{ H} \), \( R = 170 \text{ \Omega} \), and \( C = 3.5 \mu\text{C} \). The voltage amplitude of the ac source is 12 V and the \( \omega = 380 \text{ rads/s} \). Calculate the current amplitude (or peak current).

(a) 39.3 A  
(b) 0.28 A  
(c) 0.022 A  
(d) 0.017 A  
(e) 3.1 A  

\[ i = \frac{2\pi}{\omega} \text{ \frac{V}{R} \sin\omega t} \]
\[ 751.98 \text{ A} \]

\[ \varepsilon_0^2 = \Delta \varepsilon_k^2 + (\Delta \varepsilon_k \cdot \Delta n)^2 \]

\[ \varepsilon_d^2 = I^2 \left( \varepsilon + (x - x_0)^2 \right) \]
10. (6 point) A hollow metal sphere has inner radius \( R_i = 5 \text{ mm} \) and outer radius \( R_o = 10 \text{ mm} \). If the electric field at \( r = 3 \text{ mm} \) is \( 2.0 \times 10^8 \text{ V/m} \) pointing towards the center of the sphere, find the charge at the center of the sphere (if any) and the charge on the inner surface of the sphere.

\[
\sum E \cdot dA = \frac{Q_{\text{enc}}}{\epsilon_0} = \frac{4\pi R_o^2}{\epsilon_0}
\]

\[
Q_{\text{enc}} = 4\pi R_o^2 \cdot \frac{E}{\epsilon_0} = 4 \cdot 2 \times 10^6 \cdot \pi \cdot (3 \times 10^{-3})^2 \cdot 9.85 \times 10^{-12}
\]

\[
Q_{\text{enc}} = 2.00 \times 10^{-9} \text{ C}
\]

Charge on inner surface of sphere = \( +2.00 \times 10^{-9} \text{ C} \)

11. (6 points) Two point charges are placed on the x-axis. The first charge \( q_1 = 10.0 \text{ nC} \) is placed at \( x = 12.0 \text{ m} \) and the second charge \( q_2 = 6.0 \text{ nC} \) is placed at \( x = 9.0 \text{ m} \). Calculate the electric field (magnitude and direction) at the origin.

\[
E = k \frac{Q}{r^2}
\]

\[
= k \left( \frac{-Q}{r_1^2} + \frac{Q}{r_2^2} \right)
\]

\[
= \left( \frac{6.94 \times 10^{-10}}{12^2} + \frac{7.41 \times 10^{-10}}{9^2} \right)
\]

\[
E = 1.04 \text{ N/C} \text{ To Right}
\]

12. (6 points) A battery of \( \epsilon = 12.0 \text{ V} \) is connected to the circuit shown in the figure via terminals "a" and "b". Find the electrical current through the 60 \( \Omega \) resistor.

\[
\Delta V = I \cdot R
\]

\[
E = \frac{\Delta V}{R} = 0.353 \text{ Amps}
\]

\[
\Delta V = IR
\]

\[
\Delta V_{\text{bc}} = 8.47 = IR
\]

\[
I = 0.141 \text{ Amps}
\]
13. (6 points) Three electrons are on a circle of radius 3.0 mm fixed on a table. (a) Find the electric potential at the point P on the axis and 4.0 mm above the center of the circle. (b) If a fourth electron is placed at point P and released, what is its ultimate velocity of this electron?

\[ \gamma = \frac{105}{m} \]

\[ \Delta V = 4 \sum \frac{Q}{q} \]

\[ \Delta V = \frac{kQ}{\sqrt{3}} \]

\[ \Delta V = 8.64 \times 10^{-2} \text{ V} \]

\[ U = \Delta V \theta = \frac{1}{2}mv^2 = \frac{k}{3} \theta^2 \]

14. (10 points) A square coil of side \( L = 2.5 \text{ cm} \) moves into a \( \mathbf{B} \) field (1.5 Tesla and out of paper as shown) at a constant velocity \( V = 6500 \text{ m/s} \) along the positive \( x \) direction. Find the induced EMF (magnitude and direction) in the square coil when its right side is at a position \( x \) in the range of \( 0 < x < L \). (Here \( L < L_0 \) and positive direction is counterclockwise.)

\[ \mathbf{EMF} = VLB \]

\[ = (6500) \cdot (1.025) \cdot (1.5) \]

\[ \mathbf{EMF} = 2.4 \times 10^3 \text{ Volts} \] (clockwise)

\[ \oint \mathbf{B} \cdot d\mathbf{A} = \Phi \]

\[ \mathbf{BA} = \Phi \]

\[ \mathbf{BLV} = \Phi \]
15. (4 points) A current $I$ is flowing in a loop of radius $R=0.5 \text{m}$. It induces a magnetic field $B$ as shown. (a) Draw the current direction on the loop and label the north and south poles? (b) The point $P$ is on the axis of the loop at a distance of $0.5 \text{ m}$ from the center of the loop. If the magnetic field at $P$ is $1.26 \mu \text{T}$, what is the magnitude of the current in the wire?

$$B = \frac{\mu_0 I}{2\left(\sqrt{x^2 + t^2}\right)^{3/2}}$$

$$1.26 \times 10^{-6} = \frac{4\pi \times 10^{-7} \cdot I \cdot 0.5^2}{2\left(0.5^2 + 0.5^2\right)^{3/2}}$$

$$I = 1.284 \text{ amps}$$

16. (10 points) A 20 turn square coil of wire having sides of $2.0 \text{ cm}$ and resistance of $0.5 \Omega$ is placed in a $0.5 \text{ T}$ magnetic field oriented for maximum magnetic flux. The coil is connected to an uncharged $2.0 \mu \text{F}$ capacitor. What is the voltage across the capacitor after the coil is quickly pulled out of the magnetic field?

$$\mathbf{A} = B \cdot \mathbf{dA}$$

$$\mathbf{E} = \mu_0 \frac{d\mathbf{B}}{dt} = \mathbf{IE}$$

17. (6 points) A parallel plate capacitor consists of two disks of radius $5.0 \text{ cm}$ that are $1.0 \text{ mm}$ apart. The electric field is uniform between the plates and is zero outside. It is increasing at a rate of $100 \text{ V/m/s}$. Calculate magnitude of the magnetic field at a point “P” $6.0 \text{ cm}$ from the center of the disk.

$$\oint B \cdot ds = \mu_0 \left[ I + \epsilon_0 \frac{d\mathbf{E}}{dt}\right]$$

$$\mathbf{A} = \oint \mathbf{E} \cdot d\mathbf{A}$$

$$\mathbf{D} = \mathbf{E} \cdot \mathbf{A} = \mathbf{E} \cdot \pi r^2$$

$$\frac{d\mathbf{B}}{dt} = \mathbf{E} \cdot \pi r^2$$

$$B \cdot 2\pi r^2 = \mu_0 \epsilon_0 \cdot \mathbf{E} \cdot \pi r^2$$

$$B = \frac{\mu_0 \epsilon_0 \cdot \mathbf{E} \cdot \pi r^2}{2\pi r^2}$$
18. (6 points) In the RLC circuit below $E = 10.0 \text{V cos } (\omega t)$, $R=5.0 \, \Omega$, $L=5.0 \, \text{mH}$ and $C=10.0 \, \mu\text{F}$
   a) For what value of the angular frequency $\omega$ will the average power absorbed in the circuit be a maximum? 
   b) At this particular $\omega$ what is the average power absorbed by the circuit?

\[
X_L = \omega L = 22.36 \\
X_C = \frac{1}{\omega C} = 22.36 \\
\omega_0 = \frac{1}{\sqrt{LC}} \\
L = 0.5 \text{H} \\
C = 10 \times 10^{-6} \text{F} \\
\omega = \frac{4\pi + 2 \sqrt{3}}{3} \\
V_{ac} = \frac{V}{\sqrt{2}} \\
Z = \sqrt{R^2 + (X_L - X_C)^2} \\
I = 1.41 \text{ Amp} \\
V_{mm} = 7.07V \\
\Delta V = \frac{I B}{4 \pi} \\
P = \frac{I^2}{R} = 10 \text{ Watts}
\]

19. **Bonus Problem** (10 points) A solid insulating sphere with radius R carries total charge Q. The charge density inside the sphere is $\rho = \frac{C}{r}$ where $C=\frac{Q}{2\pi R^2}$. Use Gauss's law to calculate the electric field inside the sphere for $r<R$.

\[
\oint E \cdot dA = \frac{\rho_{\text{en}}}{\varepsilon_0} \\
\oint E \cdot dA = \frac{Q_{\text{en}}}{\varepsilon_0} = 5 \\
E = \frac{Q_{\text{en}}}{4\pi \varepsilon_0 r^2} \\
E = \frac{\rho_{\text{en}}}{20 \pi r^2} = \frac{P}{\varepsilon_0} \\
E = \frac{Q}{2\pi R^2 \varepsilon_0}
\]