A

Math 221

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Disposal of Waste Created by Saponification

Saponification is the process by which soap is prepared. This process begins with fat from animals or vegetable fat being reacted with substances such as potassium or sodium hydroxide to create two products. One product created is glycerol and the other is fatty acid salt called soap. These two products are separated by the precipitation of the fatty acid salt. The precipitation of soap is caused by the addition of sodium chloride to the solution. This causes the "non-soap" substances to gather at the surface of the solution. This layer formed at the top with the dissolved sodium chloride is filtered out from the top of the solution and collected as waste. Although most households obtain soap that is mass produced, this process of creating soap is very common among villages where money is scarce. However, in wealthier areas there are waste disposal ordinances on the contents of discarded fluids. If this method of making soap was done in an urban area certain calculations would be necessary to ensure that the disposal of waste would not violate ordinances set in place. In this particular problem, the maximum concentration of sodium chloride in any disposed waste must not be greater than 13g/L. The container waste is disposed into is capable of holding sixteen liters of fluid. The original concentration of sodium chloride in the fluid in this tank is 50g/L, so there are eight hundred total grams of sodium chloride in the sixteen liter solution. In order to comply with the waste disposal regulation, fresh water is pumped into the tank at a rate of two liters per minute. Also, waste sodium chloride with a concentration of 25g/L is pumped into the tank at a rate of 2L/min. The stream containing the fresh water will be referred to as Stream A, and the stream containing the waste sodium chloride will be referred to as Stream B. Also, in order to maintain the water level of the tank at sixteen liters, a concentrated stream flows out of the tank at a rate of 4L/min. In this problem it is assumed that streams A and B enter the tank at exactly the same moment, and the concentration of the tank changes instantly. The concentration of the fluid exiting the tank is represented by "x." The following equation is a mass balance equation for sodium chloride in the tank:

$$\text{Accumulation} = \text{Input} - \text{Output} + \text{Generation} - \text{Consumption}$$

Equation 1

However, in this process sodium chloride is neither generated nor consumed by any reaction in the tank because no reaction occurs. Therefore, the generation and consumption terms in Equation 1 are equal to zero. Using this information, Equation 1 is equivalent to

$$\frac{dx}{dt} = (25\text{g/L})(2.0\text{L/min}) + (0\text{g/L})(2.0\text{L/min}) - (x\ \text{g/L})(4\text{L/min})$$

Equation 2

Combining the terms in Equation 2 gives

$$\frac{dx}{dt} + 4x = 50$$

Equation 3

Before any contents from Stream A and Stream B were added to the tank the concentration was 800g/16L at t = 0. This is expressed as

$$x(0) = \frac{800g}{16L} = 50\text{g/L}$$

Equation 4

The following figure represents the flow of water in and out of the tank:

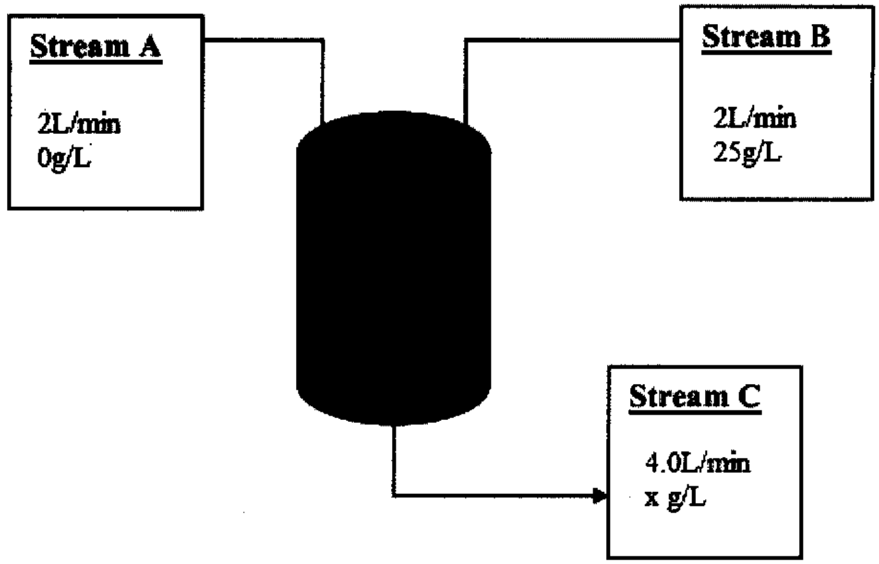

Figure 1: Flow Rate and Composition of Streams

The differential equation displayed as Equation 3 was solved using the initial value of $x(0) = 50g/L$. The following shows the process of solving the equation:

$$\frac{dx}{dt} + 4x = 50$$

$$\frac{dx}{dt} = -4x + 50$$

$$\frac{dx}{dt} = -\frac{1}{4}(x - \frac{25}{2})$$

$$\int \frac{dx}{x - \frac{25}{2}} = \int -\frac{dt}{4}$$

$$\ln|x - \frac{25}{2}| = -\frac{1}{4}t + C$$

$$x - \frac{25}{2} = K\, e^{-\frac{1}{4}t}$$

$$x(t) = K\, e^{-\frac{1}{4}t} + \frac{25}{2}$$

$$x(0) = 50 = K\, e^{0} + \frac{25}{2}$$

$$\frac{75}{2} = K$$

$$x(t) = \frac{75}{2} e^{-\frac{1}{4}t} + \frac{25}{2}$$

Equation 5

The following shows the calculations made to determine at what time "t" the waste being disposed will meet the required concentration of salt being disposed:

Find when x(t) = 13 g/L

$$13 = \frac{75}{2} e^{-\frac{1}{4}t} + \frac{25}{2}$$

$$.013 = e^{-\frac{1}{4}t}$$

$$\ln(\frac{1}{75}) = \ln(e^{-\frac{1}{4}t})$$

$$\ln(\frac{1}{75}) = -\frac{1}{4}t$$

$$t = -4\ln(\frac{1}{75}) \approx 17.27 \text{ minutes}$$

Sample Calculation 1

Therefore, after about 17 minutes the waste disposed will begin to meet the disposal ordinance for the area. Upon further examination of the system, the concentration of sodium chloride being disposed of will approach 12.5g/L. Here is a table that shows the concentration of waste leaving the tank as time increases:

Time (minutes)	Concentration of Waste Leaving the Tank (g/L)
0	50.000
5	23.244
10	15.578
15	13.382
18	12.917
20	12.753
25	12.572

Table 1: Concentration of Waste Leaving the Tank

The trend shown in Table 1 is that the concentration of sodium chloride in the waste exiting the tank will become about 12.5g/L after twenty-five minutes. The disposal of waste from the tank could be modeled by a more sophisticated equation that would be able to more precisely predict the concentration of sodium chloride in the waste being disposed from the tank. However, the relatively simple differential equation used to model this problem is more than sufficient in describing the concentration of sodium chloride in the disposed liquids. The differential equation used for this system is just one example of the many differential equations that are used to model systems in a variety of processes. It is very important to understand how to use differential equations to provide solutions for real-world problems.